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6. AUTHOR(S)

H. S. Tzou and H. Q. Fu

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Department of Mechanical Engineering
University of Kentucky
Lexington, KY 40506-0046

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Active structures possessing self sensation and action/reaction capabilities provide new design alternatives for new structural systems. In this paper, distributed piezoelectric layers coupled with elastic continua are used as *distributed* sensors and actuators for structural monitoring and control. It was noted that a fully (symmetrically) distributed piezoelectric sensor/actuator could lead to minimum, or zero, sensing/control effects for anti-symmetrical modes of structures, especially with symmetrical boundary conditions. One method to improve the performance is to segment the symmetrically distributed sensor/actuator layers into a number of colocated sub-segments. However, the effects of segmented distributed sensors/actuators are not quantitatively investigated. In this paper, distributed vibration sensing and control of continua using segmented distributed piezoelectric sensors and actuators is studied. In Part-1, mathematical models of a plate with single-piece symmetrically distributed and multi-piece segmented-distributed sensors/actuators are formulated and analytical solutions are derived. Based on analytical solutions, it proves that the singe-piece symmetrically distributed sensor/actuator layers are deficient for anti-symmetrical modes, all even modes, of the plate. The single-piece distributed sensor/actuator layers are further divided into four equally segmented pieces from which sensor and actuator pieces are colocated. Analytical solutions show that quarterly segmented distributed sensors/actuators can sense/control most of the natural modes, except for all quadruple modes.

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A STUDY ON SEGMENTATION OF DISTRIBUTED PIEZOELECTRIC SENSORS AND ACTUATORS: PART 1 – THEORETICAL ANALYSIS†

H. S. Tzou and H. Q. Fu
Department of Mechanical Engineering
Center for Robotics and Manufacturing Systems
University of Kentucky
Lexington, Kentucky

ABSTRACT

Active structures possessing self sensation and action/reaction capabilities provide new design alternatives for new structural systems. In this paper, distributed piezoelectric layers coupled with elastic continua are used as *distributed* sensors and actuators for structural monitoring and control. It was noted that a fully (symmetrically) distributed piezoelectric sensor/actuator could lead to minimum, or zero, sensing/control effects for anti–symmetrical modes of structures, especially with symmetrical boundary conditions. One method to improve the performance is to segment the symmetrically distributed sensor/actuator layers into a number of colocated sub–segments. However, the effects of segmented distributed sensors/actuators are not quantitatively investigated. In this paper, distributed vibration sensing and control of continua using segmented distributed piezoelectric sensors and actuators is studied. In Part–1, mathematical models of a plate with single–piece symmetrically distributed and multi–piece segmented–distributed sensors/actuators are formulated and analytical solutions are derived. Based on analytical solutions, it proves that the singe–piece symmetrically distributed sensor/actuator layers are deficient for anti–symmetrical modes, all even modes, of the plate. The single–piece distributed sensor/actuator layers are further divided into four equally segmented pieces from which sensor and actuator pieces are colocated. Analytical solutions show that quarterly segmented distributed sensors/actuators can sense/control most of the natural modes, except for all quadruple modes.

INTRODUCTION

In recent years there are significant interests and efforts trying to integrate active materials (such as piezoelectrics, shape–memory alloys, electrostrictive materials, magnetostrictive materials, electrorheological fluids, etc.) with an elastic structure such that the structure transforms from a completely passive system to an active adaptive system (Tzou, 1991a). With the rapid development of VLSI technologies, adding an "intelligence" to the structure could also become a reality in the near future (Tzou & Fukuda, 1991). In the development of active piezoelectric/elastic structures, it was observed that symmetrically distributed piezoelectric sensors and actuators have observability and controllability deficiencies in monitoring and controlling continua, especially with symmetrical boundary conditions. With a single–piece symmetrically distributed sensor and actuator, anti–symmetrical structural modes may not be observable and controllable because positive and negative sensing/control signals on different regions of the continua could cancel out each other. One method to improve the controllability and observability of distributed piezoelectric sensors/actuators is to segment them into a number of smaller pieces – sub–areas. However, the effects of these segmented distributed sensors/actuators have not been quantitatively analyzed. Thus, distributed vibration sensing and control of plates using segmented sensors and actuators are investigated in this study.

Theories on distributed sensing and control of shell continua using distributed piezoelectric layers were proposed recently. Applications to plate structures were also demonstrated in case studies (Tzou, 1991b; Tzou & Tseng, 1991). Ricketts studied a piezoelectric polymer flexural plate hydrophone (1981) and the frequency of completely free composite piezoelectric plates (1989). Lee (1990) proposed a theory for a laminated piezoelectric plates with applications to distributed sensor/actuator designs. His formulations suggested that the distributed piezoelectric layers are capable of sensing and controlling bending, shearing, shrinking, and stretching effects of a plate. Burke and Hubbard (1990) studied distributed transducer control designs for thin plates with general boundary conditions. Modal control and observation deficiencies were also exploited. Dimitriadis, et al. (1991) investigated distributed vibration excitations of thin plates using piezoelectric actuators. In this paper, distributed sensing and control (velocity and displacement feedback) of a plate is derived from the generic distributed sensing and control theories of thin shells (Tzou, 1991b). Modal sensing and control of the plate is derived using the modal expansion method and equivalent line control moments are also derived in the modal domain. Observation and control deficiencies of symmetrically distributed single–piece and segmented sensors/actuators are proved based on analytical solutions. Advantages of using segmented distributed sensors/actuators are discussed. (Emphasis is placed on the evaluation of distributed sensor/actuator segmentation.)

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MATHEMATICAL MODELING

In general, closed–loop system equations, in three principle directions, of a generic shell continuum coupled with distributed piezoelectric sensors and actuators can be expressed in a simplified form (Tzou, 1991a&b):

$$L_i(u_1,u_2,u_3) - \rho h\ddot{u}_i = - F_i - L_i^a(M_{ij}^a,N_{ij}^a)\ , i = 1,2,3,\quad (1)$$

where L_i denotes all derivative operations of the elastic components; u_i is the displacement; ρ is the mass density; h is the shell thickness; F_i is the mechanical excitation; L_i^a denotes the derivatives of the induced control membrane forces and bending moments; and M_{ij}^a and N_{ij}^a are the control moments and membrane forces on the ith plane in the jth direction. For an uncoupled transverse vibration of a thin elastic shell, the generic system equation is reduced to

$$L_3(u_3) - \rho h\ddot{u}_3 = - F_3 - L_3^a(M_{ij}^a)\ .\quad (2)$$

If the shell continuum has an inherited viscous damping, the system equation can be further expressed as (Soedel, 1981):

$$L_3(u_3) - \rho h\ddot{u}_3 - c\dot{u} = - F_3 - L_3^a(M_{ij}^a)\ ,\quad (3)$$

where c is the *equivalent viscous damping factor*. In the case of piezoelectricity induced damping, the control counteracting force/moment is usually assumed to be proportional to the velocity. For a free vibration analysis (eigenvalue analysis), all external excitations (both mechanical and feedback) and damping forces are zeros, i.e., $F_i = 0$, $L_i^a(M_{ij}^a,N_{ij}^a) = 0$, and $c\dot{u} = 0$. It is also assumed that all points on the continuum oscillate harmonically at one of the natural frequencies, i.e., $u_i(x,y) = U_i(x,y)\, e^{j\omega t}$. Thus, one can derive

$$L_i(U_{1mn},U_{2mn},U_{3mn}) + \rho h\omega_{mn}^2 U_{imn} = 0\ ,\quad (4)$$

where ω_{mn} is the natural frequency of the mn–th mode and $U_{imn}(\alpha_1,\alpha_2)$ is the mode shape function – a spatial function of coordinates.

Plate with Distributed Sensor and Actuator

A rectangular plate with two biaxial oriented piezoelectric polymeric layers, one serves as a distributed sensor and the other a distributed actuator, is used as a physical system in this study, Figure 1. Segmentation of distributed sensors and actuators will be discussed later. (Note that which piezoelectric layer serves as a sensor or actuator is not crucial. In this case, the bottom layer serves as a sensor and the top an actuator.) The piezoelectric layers are assumed to be perfectly bonded on the surfaces of the plate and the physical properties of the bonding material are neglected. It is assumed that the transverse bending oscillation dominates the plate motion i.e., the in–plane membrane oscillations are neglected, in the later analyses.

It is assumed that the in–plane twisting piezoelectric constant is insignificant, i.e., $M_{xy}^a = 0$. The piezoelectric layers

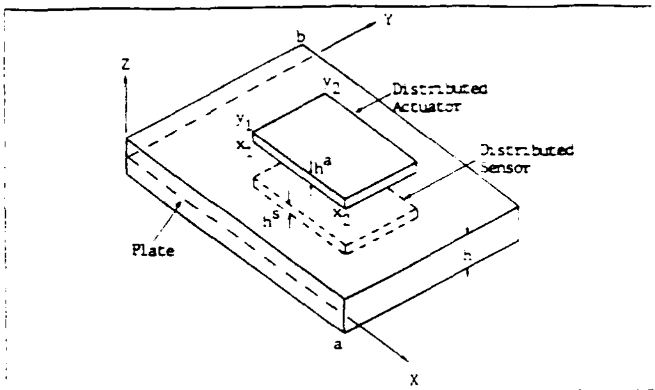

Fig.1 A plate with distributed sensor and actuator layers.

(h^s and h^a) are much thinner than the plate thickness h; thus, the effect of piezoelectric layer thickness is neglected in the analyses. The system equation for the plate with distributed sensor and actuator layers can be written as

$$D\left[\frac{\partial^4 u_3}{\partial x^4} + 2\frac{\partial^4 u_3}{\partial x^2\partial y^2} + \frac{\partial^4 u_3}{\partial y^4}\right] + \rho h\ddot{u}_3 + c\dot{u}_3 = F_3 + \frac{\partial^2 M_{xx}^a}{\partial x^2} + \frac{\partial^2 M_{yy}^a}{\partial y^2}\ ,\quad (5)$$

where D is the bending stiffness, $D = (Yh^3)/[12(1-\mu^2)]$; Y is Young's modulus and μ is Poisson's ratio. M_{xx}^a and M_{yy}^a are distributed control moments (Tzou, 1991b) and they can be expressed in a modal expansion form (Tzou, 1992):

$$M_{xx}^a = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \eta_{mn}(t)\, r_1^a d_{31} Y_p \mathcal{G}\, \phi_{mn}^s\ ,\quad (6)$$

$$M_{yy}^a = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \eta_{mn}(t)\, r_2^a d_{32} Y_p \mathcal{G}\, \phi_{mn}^s\ .\quad (7)$$

Note that η_{mn} is the modal participation factor; r_i^a denotes the moment arm, a distance measured from the neutral surface to the mid–plane of the actuator and $r_i^a = (h + h_i^a)/2$; d_{3i} is the piezoelectric strain constant, Y_p is Young's modulus of the piezoelectric layer, and $\mathcal{G}$ is the feedback gain. ϕ_{mn}^s is a feedback voltage which could be a reference voltage (open–loop) or a sensor voltage (closed–loop) and it is assumed to be a spatial function. In the case of closed–loop feedback, ϕ_{mn}^s is the mn–th *unit modal sensing signal* of a distributed piezoelectric sensor expressed as a function of mode shape function.

$$\phi_{mn}^s = - h^s\left[h_{31}r_1^s\left(\frac{\partial^2 U_{3mn}}{\partial x^2}\right) + h_{32}r_2^s\left(\frac{\partial^2 U_{3mn}}{\partial y^2}\right)\right]\ ,\quad (8\text{–a})$$

$$\phi_{mn}^s = - \frac{h^s}{A^s}\int_{A^s}\left[h_{31}r_1^s\frac{\partial^2 U_{3mn}}{\partial x^2} + h_{32}r_2^s\frac{\partial^2 U_{3mn}}{\partial y^2}\right]dA^s\ ,\quad (8\text{–b})$$

where h^S is the thickness of the distributed piezoelectric sensor layer; A^S is the effective electroded sensor area; h_{31} and h_{32} are the piezoelectric constants; r_i^S denotes the distance measured from the neutral surface to the mid-plane of the sensor layer and $r_i^S = (h + h_i^S)/2$. Eq.(8–a) denotes the spatial distribution and Eq.(8–b) denotes the averaged signal output.

Modal Expansion and Vibration Controls

Based on the *modal expansion technique*, the dynamic response of a distributed system can be represented by a summation of the responses of all participating modes, i.e., $u_3(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \eta_{mn}(t)\, U_{3mn}(x,y)$, where U_{3mn} is the unit spatial part (*mode shape function*) and η_{mn} is the temporal part – an amplitude factor called the *modal participation factor*. Substituting the modal expression into the system equation results in an equation in terms of modal participation factors. Using the modal orthogonality of natural modes, one can derive (Fu, 1990)

$$\ddot{\eta}_{mn} + \frac{c}{\rho h}\dot{\eta}_{mn} + \omega_{mn}^2\eta_{mn} = \hat{F}_k + \frac{1}{\rho h N_k}\int_x\int_y \left[\frac{\partial^2 M_{xx}^a}{\partial x^2} + \frac{\partial^2 M_{yy}^a}{\partial y^2}\right] U_{3mn}\,dxdy\,, \tag{9}$$

where

$$\hat{F}_k = \frac{1}{\rho h N_k}\int_x\int_y F_3 U_{3mn}\,dxdy\,, \tag{10}$$

and N_k is defined by the mode shape functions:

$$N_k = \int_x\int_y U_{3mn}^2\,dxdy\,. \tag{11}$$

(Note that this generic expression is only for transverse vibration modes.) For a simply supported plate with a dimension of a×b, the transverse mode shape function is

$$U_{3mn} = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,. \tag{12}$$

Substituting U_{3mn} into N_k and carrying out the surface integration, one can derive

$$N_k = \frac{ab}{4}\,. \tag{13}$$

Using the definition of damping ratio for a single degree of freedom system, one can rewrite the modal coordinate equation as

$$\ddot{\eta}_{mn} + 2\zeta_{mn}\omega_{mn}\dot{\eta}_{mn} + \omega_{mn}^2\eta_{mn} = \hat{F}_k + \frac{1}{\rho h N_k}\int_x\int_y \left(\frac{\partial^2 M_{xx}^a}{\partial x^2} + \frac{\partial^2 M_{yy}^a}{\partial y^2}\right) U_{3mn}\,dxdy\,, \tag{14}$$

where

$$\zeta_{mn} = \frac{c}{2\rho h\omega_{mn}}\,. \tag{15}$$

is the *modal damping ratio*. Substituting the modal expressions of control bending moments into the modal equation yields

$$\ddot{\eta}_{mn} + 2\zeta_{mn}\omega_{mn}\dot{\eta}_{mn} + \omega_{mn}^2\eta_{mn} = \hat{F}_k + \frac{1}{\rho h N_k}\int_x\int_y \left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta_{mn}\left(\frac{\partial^2 M_{xmn}}{\partial x^2} + \frac{\partial^2 M_{ymn}}{\partial y^2}\right)\right] \cdot U_{3mn}\,dxdy\,, \tag{16}$$

where M_{xmn} and M_{ymn} are the *unit modal control moments*:

$$M_{xmn} = r_1^a d_{31} Y_p \mathcal{G}\, \phi_{mn}^S\,, \tag{17}$$

$$M_{ymn} = r_2^a d_{32} Y_p \mathcal{G}\, \phi_{mn}^S\,. \tag{18}$$

Since the time function of feedback control input is also a function of system modal participation factors, the common terms, the mn–th term in the summations,

$$\frac{1}{\rho h N_k}\eta_{mn}\int_x\int_y \left[\frac{\partial^2 M_{xmn}}{\partial x^2} + \frac{\partial^2 M_{ymn}}{\partial y^2}\right] U_{3mn}\,dxdy\,. \tag{19}$$

represents the control effect of the mn–th mode; and it can be moved to the left side and combined with the appropriate system term, e.g., η_{mn} for a *displacement feedback* or $\dot{\eta}_{mn}$ for a *velocity feedback*. (These two feedback controls will be discussed shortly.) All the other terms within the summations represent the cross coupling control effects from all other residual modes. Define a *modal feedback factor* $\hat{M}_{mn}$ which is independent of amplitude (Fu, 1990):

$$\hat{M}_{mn} = \frac{1}{\rho h N_k}\int_x\int_y \left[\frac{\partial^2 M_{xmn}}{\partial x^2} + \frac{\partial^2 M_{ymn}}{\partial y^2}\right] U_{3mn}\,dxdy\,; \tag{20}$$

and use $\hat{T}_k$ to denote the cross coupling feedback effects due to residual modes,

$$\hat{T}_k = \frac{1}{\rho h N_k}\sum_{\substack{p=1\\p\neq m}}^{\infty}\sum_{\substack{q=1\\q\neq n}}^{\infty}\eta_{pq}\int_x\int_y \left[\frac{\partial^2 M_{xpq}}{\partial x^2} + \frac{\partial^2 M_{ypq}}{\partial y^2}\right] \cdot U_{3mn}\,dxdy\,. \tag{21}$$

1) Displacement Feedback Control

For a *displacement feedback* proportional control, the modal equation can be rewritten as

$$\ddot{\eta}_{mn} + 2\zeta_{mn}\omega_{mn}\dot{\eta}_{mn} + (\omega_{mn}^2 - \hat{M}_{mn})\eta_{mn} = \hat{F}_k + \hat{T}_k\,, \tag{22}$$

where

$$\hat{T}_k = \frac{1}{\rho h N_k}\sum_{\substack{p=1\\p\neq m}}^{\infty}\sum_{\substack{q=1\\q\neq n}}^{\infty}\eta_{pq}\int_x\int_y \left[\frac{\partial^2 M_{xpq}}{\partial x^2} + \frac{\partial^2 M_{ypq}}{\partial y^2}\right] \cdot U_{3mn}\,dxdy\,, \tag{23}$$

$\hat{F}_k = \frac{1}{\rho h N_k}\int_x\int_y F_3 U_{3mn}\,dxdy$ and $N_k = \int_x\int_y U_{3mn}^2\,dxdy$. Both are defined for the transverse vibration modes u_3 only.

2) Velocity Feedback Control

For a *velocity feedback* control (derivative control), the feedback time function is a first derivative of modal participation factor, and the induced moments become

$$M^a_{xx} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \dot{\eta}_{mn}(t)\, M_{xmn}\,, \tag{24}$$

$$M^a_{yy} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \dot{\eta}_{mn}(t)\, M_{ymn}\,. \tag{25}$$

Moving the common terms to the left and combining them with $\dot{\eta}_{mn}$ give

$$\ddot{\eta}_{mn} + (2\zeta_{mn}\omega_{mn} - \hat{M}_{mn})\dot{\eta}_{mn} + \omega^2_{mn}\eta_{mn} = \hat{F}_k + \hat{T}'_k\,, \tag{26}$$

where

$$\hat{T}'_k = \frac{1}{\rho h N_k} \sum_{\substack{p=1\\ p\neq m}}^{\infty} \sum_{\substack{q=1\\ q\neq n}}^{\infty} \dot{\eta}_{pq} \int_x \int_y \left[\frac{\partial^2 M_{xpq}}{\partial x^2} + \frac{\partial^2 M_{ypq}}{\partial y^2}\right] \cdot U_{3mn}\, dx dy\,. \tag{27}$$

Since the modal feedback factor $\hat{M}_{mn}$ is independent of the vibration amplitude, it can be treated as a system parameter. Once it is defined, the new system parameter can be estimated without solving for a particular modal participation factor $\eta_{mn}(t)$ completely.

Note that the above evaluation procedure is in a generic form. It can be employed for other geometric configurations whose mode shape functions, either exact or approximate, are known. These configurations include beams with most common boundary conditions, some plates, simply supported cylindrical panels or cylinders, and some other shell structures (Soedel, 1981).

ONE–PIECE SYMMETRICALLY DISTRIBUTED SENSOR AND ACTUATOR

In a closed–loop feedback system, the induced distributed control moments are originally initiated from the distributed sensor signal. In this section, formulation of a distributed sensing signal and distributed control moments for a simply supported plate with a single distributed piezoelectric sensor and a actuator is presented. Note that a simply supported plate is used in this study. (Segmentation of distributed sensor and actuator layers will be discussed in the next section.)

Distributed Sensing

The mn–th mode shape function $U_{3mn}(x,y)$, in the transverse direction, for a simply supported rectangular plate is given by $U_{3mn}(x,y) = \sin\frac{m\pi x}{a}\, \sin\frac{n\pi y}{b}$. The unit modal sensor signal (averaged) is $\phi^s_{mn} = -\frac{h^s}{A^s}\int_{A^s}\left[h_{31}r^s_1(\partial^2 U_{3mn}/\partial x^2) + h_{32}r^s_2(\partial^2 U_{3mn}/\partial y^2)\right] dA^s$. It is assumed that the sensor layer is fully distributed on the plate surface. Substituting the mode shape function into the sensor equation and integrating over whole sensor surface, x = (0, a) and y = (0, b), one can derive

$$\phi^s_{mn} = \frac{h^s}{mn}\left[h_{31}r^s_1\left(\frac{m}{a}\right)^2 + h_{32}r^s_2\left(\frac{n}{b}\right)^2\right](1-\cos m\pi) \cdot (1-\cos n\pi)\,. \tag{28}$$

Define a sensitivity constant S_{mn} for the mn–th mode:

$$S_{mn} = \frac{h^s}{mn}\left[h_{31}r^s_1\left(\frac{m}{a}\right)^2 + h_{32}r^s_2\left(\frac{n}{b}\right)^2\right], \tag{29}$$

so that the sensor equation can be simplified to

$$\phi^s_{mn} = S_{mn}(1-\cos m\pi)(1-\cos n\pi)\,. \tag{30}$$

Note that the output signal vanishes, i.e., zero output, if the mode order m or n is an even number. This observability problem will be further discussed later. Additional feedback controllability problem induced by the constant voltage will be answered in the next section.

Distributed Vibration Control

It is assumed that the distributed piezoelectric actuator layer covers the plate from locations x_1 to x_2 and from y_1 to y_2, Figure 1. (Note that $x_1 = 0$, $x_2 = a$, $y_1 = 0$, and $y_2 = b$ for a fully distributed actuator layer.) Since the feedback voltage is constant over whole actuator surface if the electrode resistance ignored, the induced control moment is also uniformly distributed on the actuator covered area. As discussed previously, the moment function can be separated into a temporal function part and a spatial function part: $M^a_{xx} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \eta_{mn}(t) M_{xmn}$ and $M^a_{yy} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \eta_{mn}(t) M_{ymn}$. Thus, one can express the moment spatial distribution using a unit step function u_s:

$$u_s(x - x_i) = \begin{cases} 1 & \text{for } x > x_i \\ 0 & x \le x_i \end{cases}. \tag{31}$$

Then the distributed control moments M^*_{imn}'s can be expressed as

$$M^*_{xmn} = M_{xmn}[u_s(x-x_1)-u_s(x-x_2)][u_s(y-y_1)-u_s(y-y_2)]\,, \tag{32}$$

$$M^*_{ymn} = M_{ymn}[u_s(x-x_1)-u_s(x-x_2)][u_s(y-y_1)-u_s(y-y_2)]\,, \tag{33}$$

where the magnitudes for proportional feedback M_{xmn} and M_{ymn} were defined previously. Substituting the sensor signal of the distributed sensor into the unit modal control moments yields

$$M_{xmn} = r^a_1 d_{31} Y_p \mathcal{G}\, S_{mn}(1-\cos m\pi)(1-\cos n\pi)\,, \tag{34}$$

$$M_{ymn} = r^a_2 d_{32} Y_p \mathcal{G}\, S_{mn}(1-\cos m\pi)(1-\cos n\pi)\,. \tag{35}$$

Equivalent Line Moment

Note that a uniform moment distribution is characterized as a resultant bending phenomenon which can be equated by a set of couples or moments acting at both ends of the distribution. Using an equivalent external distributed moment approximation and the modal expansion technique, one can derive a modal equation in terms of modal participation factor as

$$\ddot{\eta}_{mn} + 2\zeta_{mn}\omega_{mn}\dot{\eta}_{mn} + \omega_{mn}^2\eta_{mn}$$
$$= \hat{F}_k + \frac{1}{\rho h N_k}\int_x\int_y \left[\frac{\partial T_{11}}{\partial x} + \frac{\partial T_{22}}{\partial y}\right] U_{3mn} dxdy \,, \quad (36)$$

where T_{ii} is the distributed moment acting in the ith–direction with ~~a~~ unit $N \cdot m/m^2$ (Soedel, 1981). Comparing this equation with Eq.(14) gives

$$\frac{\partial M_{xx}^a}{\partial x} = T_{11} \,, \quad (37)$$

$$\frac{\partial M_{yy}^a}{\partial y} = T_{22} \,. \quad (38)$$

Using the identity: $\frac{d}{dx}[u_s(x - x_i)] = \delta(x - x_i)$ where $\delta(x)$ is a Dirac delta function defined as

$$\delta(x - x_i) = \begin{cases} 1 & \text{for } x = x_i \\ 0 & x \neq x_i \end{cases} \,, \quad (39)$$

which has a dimension (1/m) (Soedel, 1976), one can derive the equivalent line control moments in the modal domain:

$$\frac{\partial M_{xx}^a}{\partial x} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \eta_{mn} M_{xmn}[\delta(x-x_1) - \delta(x-x_2)][u_s(y-y_1) - u_s(y-y_2)] = T_{11} \,, \quad (40)$$

$$\frac{\partial M_{yy}^a}{\partial y} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \eta_{mn} M_{ymn}[u_s(x-x_1) - u_s(x-x_2)][\delta(y-y_1) - \delta(y-y_2)] = T_{22} \,. \quad (41)$$

The above two equations imply that in each direction the uniform moment is equivalent to two external *equivalent distributed line moments* acting at both ends of the distribution. The units of two moment definitions are consistent with each other. Thus, the equivalent line moments M^*_{imn}'s representing the control moment effects on the boundaries of actuator distribution, fully or partially distributed, are used. (Lee (1990), Burke/Hubbard (1990), and Dimitriadis, et al. (1991) also suggested the line–moment approximation in global coordinates based on their respective studies.) Modal controllability and observability of the single–piece distributed sensor and actuator, fully or partially distributed, will be emphasized in the later analyses.

Modal Feedback Factor and Modal Equations

Using the *equivalent distributed line moment* concepts, one can redefine the *modal feedback factor* $\dot{M}_{mn}$ as

$$\dot{M}_{mn} = \frac{1}{\rho h N_k}\int_x\int_y \left[\frac{\partial^2 M^*_{xmn}}{\partial x^2} + \frac{\partial^2 M^*_{ymn}}{\partial y^2}\right] U_{3mn} dxdy \,, \quad (42)$$

where M^*_{xmn} and M^*_{ymn} are the *equivalent distributed line moments*. Depending on feedback algorithms, this modal feedback factor may contribute to either damping (velocity feedback) or elasticity (displacement feedback) in the mn–th modal coordinate equation. Substituting Eqs.(32), (40) and mode shape function into the *modal feedback factor* $\dot{M}_{mn}$ and integrating the two moment terms respectively, one can obtain

$$\int_x\int_y \frac{\partial^2 M^*_{xmn}}{\partial x^2} U_{3mn} dxdy = \int_0^a\int_0^b M_{xmn}\frac{\partial}{\partial x}[\delta(x - x_1) - \delta(x - x_2)] \cdot u_s(y - y_1) - u_s(y - y_2)] \cdot \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\, dxdy \,. \quad (43)$$

Integrating by parts gives

$$\int_x\int_y \frac{\partial^2 M^*_{xmn}}{\partial x^2} U_{3mn} dxdy = -M_{xmn}\frac{mb}{na}\left[\cos\frac{m\pi x_1}{a} - \cos\frac{m\pi x_2}{a}\right]\left[\cos\frac{n\pi y_1}{b} - \cos\frac{n\pi y_2}{b}\right] . \quad (44)$$

Similarly,

$$\int_x\int_y \frac{\partial^2 M^*_{ymn}}{\partial y^2} U_{3mn} dxdy = -M_{ymn}\frac{na}{mb}\left[\cos\frac{m\pi x_1}{a} - \cos\frac{m\pi x_2}{a}\right]\left[\cos\frac{n\pi y_1}{b} - \cos\frac{n\pi y_2}{b}\right] . \quad (45)$$

Then, the *modal feedback factor* $\dot{M}_{mn}$ for the equivalent boundary line control moments of a simply supported plate can be expressed as

$$\dot{M}_{mn} = \frac{-1}{\rho h N_k}\left(M_{xmn}\frac{mb}{na} + M_{ymn}\frac{na}{mb}\right)\left[\cos\frac{m\pi x_1}{a} - \cos\frac{m\pi x_2}{a}\right] \cdot \left[\cos\frac{n\pi y_1}{b} - \cos\frac{n\pi y_2}{b}\right] . \quad (46)$$

Note again that $\dot{M}_{mn}$ vanishes for all even modes if x_1 and x_2 or y_1 and y_2 are symmetrically located about the mid–span. As discussed previously, the modal feedback factor can be combined with the modal damping term in the <u>velocity feedback</u>:

$$\ddot{\eta}_{mn} + (2\zeta_{mn}\omega_{mn} - \dot{M}_{mn})\dot{\eta}_{mn} + \omega_{mn}^2\eta_{mn} = \hat{F}_k + \hat{T}'_k \,, \quad (47)$$

where $\dot{M}_{mn}$ represents the distributed control effect and $\hat{T}'_k$, Eq.(27), denotes the cross coupling effects resulting from all other participating modes due to the closed–loop feedback. Define a modified modal damping ratio ζ'_{mn} as

$$\zeta'_{mn} = \zeta_{mn} - \frac{\dot{M}_{mn}}{2\omega_{mn}} \,, \quad (48)$$

where the system inherent modal damping ratio ζ_{mn} is assumed known from laboratory experiments. Then, the modified mn–th modal equation becomes

$$\ddot{\eta}_{mn}+2\zeta'_{mn}\omega_{mn}\dot{\eta}_{mn}+\omega_{mn}^2\eta_{mn}=\hat{F}_k+\hat{T}'_k. \qquad (49)$$

Note that $\hat{T}'_k$ is the coupling terms from the residual modes.

Modal Controllability and Observability

As discussed previously, the derived sensing and control equations suggest that the single–piece symmetrically distributed piezoelectric sensor and actuator have deficiencies in sensing and controlling all even modes – anti–symmetrical modes – of the simply supported plate. That is there are severe problems on observability and controllability for the even–order modes. According to the mode shape $\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$, the sensor layer is stretched at some locations while the other locations compressed so that the charge signs can be different. The resultant charge is zero due to those equal magnitude negative and positive instant charges.

Even if the observability problem could be solved, say by other sensing techniques, there is still modal controllability problem for the single–piece symmetrically distributed actuator. For example, when m = 1 and n = 1, the moments at both ends counteract the motion resulting in significant control effects. However, when either m or n = 2, only a moment at one end suppresses the motion, while the other augments the motion. As a result, only very minimal, or zero, control effect is generated.

In order to ensure controllabilities for most of the modes, e.g., both even and odd modes, distributed piezoelectric sensors and actuators need to be redesigned and/or new control strategies developed. One method to improve the sensing and control performance is to segment the distributed sensor and actuator layers into a number of sub–areas so that the charge/voltage cancellation problems can be minimized. As an example, the single–piece distributed sensor and actuator layers are equally divided into four smaller pieces in which sensor signals are fed back into their colocated actuator layers. In the next section, the performance of these segmented distributed sensors and actuators is evaluated using analytical techniques.

SEGMENTATION OF DISTRIBUTED SENSORS AND ACTUATORS

As discussed in the previous section, theoretical derivation suggests that there are observability and controllability problems for anti–symmetrical modes if the single–piece symmetrically distributed sensor and actuator layers are used. One method to overcome this problem is to segment both the distributed piezoelectric sensor and actuator into colocated subsections or sub–areas. That is each pair of sensor and actuator consists of top and bottom pieces of layers at the same location of the plate respectively. Then each segmented sensor can detect the local motion state. The processed sensing signal is fed back into the colocated distributed actuator resulting in a localized control effect for that sub–area only. Detailed formulation of a segmented distributed piezoelectric sensor/actuator design for the simply supported plate is presented in this section (Fu, 1990).

Segmented Distributed Sensors

It is assumed that the distributed piezoelectric sensor is divided into four equally sized segments, i.e., cut along the center lines. Figure 2 illustrates this segmentation. The four segmented sensors still cover the whole surface of the plate. A small gap is left between the two adjacent sensor segments to prevent them from short circuitry, but it is ignored in the mathematical model due to its smallness. Then each sensor segment responds to the local motion state and generates a signal output.

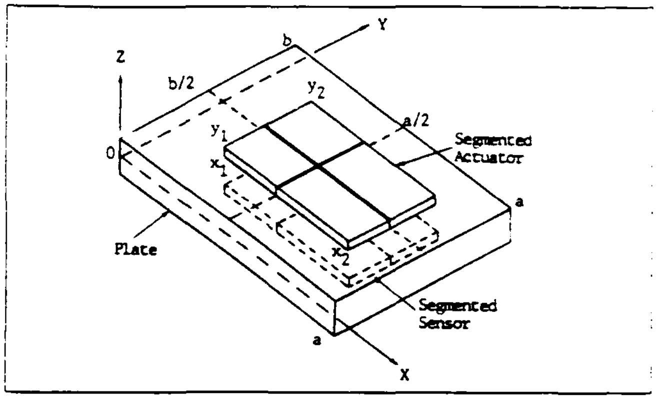

Fig.2 Segmentation of distributed piezoelectric sensors and actuators.

For the mn–th mode, segment–1 sensor outputs a signal ϕ_{mn}^{s1} as

$$\phi_{mn}^{s1}=\frac{h^s}{A^{s1}}\left[h_{31}r_1^s\left(\frac{m\pi}{a}\right)^2+h_{32}r_2^s\left(\frac{n\pi}{b}\right)^2\right]$$
$$\cdot\int_0^{\frac{a}{2}}\int_0^{\frac{b}{2}}\sin\frac{m\pi x}{a}\sin\frac{m\pi y}{b}dxdy$$
$$=4S_{mn}\left(1-\cos\frac{m\pi}{2}\right)\left(1-\cos\frac{n\pi}{2}\right), \qquad (50)$$

where S_{mn} is a sensitivity constant for the mn–th mode, i.e., $S_{mn}=\frac{h^s}{mn}\left[h_{31}r_1^s\left(\frac{m}{a}\right)^2+h_{32}r_2^s\left(\frac{n}{b}\right)^2\right]$. $r_i^s=(h+h^s)/2$. And the other three segmented sensors provide output signals as

$$\phi_{mn}^{s2}=4S_{mn}\left(1-\cos\frac{m\pi}{2}\right)\left(\cos\frac{n\pi}{2}-\cos n\pi\right), \qquad (51)$$

$$\phi_{mn}^{s3}=4S_{mn}\left(\cos\frac{m\pi}{2}-\cos m\pi\right)\left(\cos\frac{n\pi}{2}-\cos n\pi\right), \qquad (52)$$

$$\phi_{mn}^{s4}=4S_{mn}\left(\cos\frac{m\pi}{2}-\cos m\pi\right)\left(1-\cos\frac{n\pi}{2}\right). \qquad (53)$$

Note that the output signals won't vanish for most modes except for quadruples of n or m modes. Note that if n= m = 1, $\phi_{mn}^{s1}=\phi_{mn}^{s2}=\phi_{mn}^{s3}=\phi_{mn}^{s4}$. If n = m = 2, $\phi_{mn}^{s1}=\phi_{mn}^{s3}$ and $\phi_{mn}^{s2}=\phi_{mn}^{s4}=-\phi_{mn}^{s1}$.

Segmented Distributed Actuators

It is assumed that a distributed piezoelectric layer covers the center part of the plate from x_1 to x_2 and from y_1 to y_2 and it is further equally divided into four segmented distributed actuators as shown in Figure 2. According to the sign changes

of sensor outputs at different modes. Eqs (50) to (53), the signs of feedback voltages to each segmented actuator varies and so the distributions of induced moments. Thus, the control moment distributions can be written in the form of step function u_s's as

$$
\begin{aligned}
M^*_{xmn} &= M_{xmn}\Big[u_s(x-x_1) - u_s(x-\tfrac{a}{2}) - (-1)^m u_s(x-\tfrac{a}{2}) \\
&+ (-1)^m u_s(x-x_2)\Big]\Big[u_s(y-y_1) - u_s(y-\tfrac{b}{2}) - (-1)^n u_s(y-\tfrac{b}{2}) \\
&+ (-1)^n u_s(y-y_2)\Big] ,
\end{aligned}
\tag{54}
$$

$$
\begin{aligned}
M^*_{ymn} &= M_{ymn}\Big[u_s(x-x_1) - u_s(x-\tfrac{a}{2}) - (-1)^m u_s(x-\tfrac{a}{2}) \\
&+ (-1)^m u_s(x-x_2)\Big]\Big[u_s(y-y_1) - u_s(y-\tfrac{b}{2}) - (-1)^n u_s(y-\tfrac{b}{2}) \\
&+ (-1)^n u_s(y-y_2)\Big] .
\end{aligned}
\tag{55}
$$

Note that $(-1)^{m/n}$ is used for sign changes. The directions of control moments depending on mode-numbers m and n are consistent with the sign changes of signals discussed previously. Figure 3 illustrates the control moments contributed by segmented actuators for two typical modes.

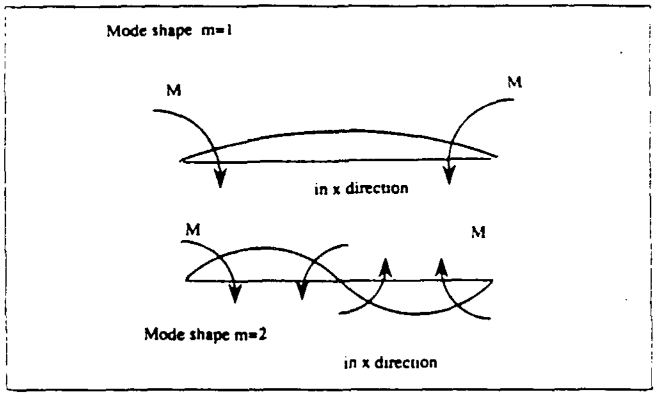

Fig.3 Moment actuation of segmented actuators.

Substituting the above two generic moment expressions into Eq.(20) for modal feedback factor and integrating each term respectively, one can derive

$$
\begin{aligned}
\int_x\!\int_y \frac{\partial^2 M^*_{xmn}}{\partial x^2} U_{3mn}\,dxdy &= -M_{xmn}\frac{mb}{na}\Big[\cos\frac{m\pi x_1}{a} - \cos\frac{m\pi}{2} \\
&- (-1)^m\cos\frac{m\pi}{2} + (-1)^m\cos\frac{m\pi x_2}{a}\Big]\Big[\cos\frac{n\pi y_1}{b} - \cos\frac{n\pi}{2} \\
&- (-1)^n\cos\frac{n\pi}{2} + (-1)^n\cos\frac{n\pi y_2}{b}\Big] ,
\end{aligned}
\tag{56}
$$

$$
\begin{aligned}
\int_x\!\int_y \frac{\partial^2 M^*_{ymn}}{\partial y^2} U_{3mn}\,dxdy &= -M_{ymn}\frac{na}{mb}\Big[\cos\frac{m\pi x_1}{a} - \cos\frac{m\pi}{2} \\
&- (-1)^m\cos\frac{m\pi}{2} + (-1)^m\cos\frac{m\pi x_2}{a}\Big]\Big[\cos\frac{n\pi y_1}{b} - \cos\frac{n\pi}{2} \\
&- (-1)^n\cos\frac{n\pi}{2} + (-1)^n\cos\frac{n\pi y_2}{b}\Big] .
\end{aligned}
\tag{57}
$$

Thus, the modal feedback factor $\dot{M}_{mn}$ for the four-piece segmented actuator configuration becomes

$$
\begin{aligned}
\dot{M}_{mn} &= \frac{-1}{\rho h N_k}\Big\{M_{xmn}\frac{mb}{na} + M_{ymn}\frac{na}{mb}\Big\} \\
&\cdot\Big[\cos\frac{m\pi x_1}{a} - \cos\frac{m\pi}{2} - (-1)^m\cos\frac{m\pi}{2} + (-1)^m\cos\frac{m\pi x_2}{a}\Big] \\
&\cdot\Big[\cos\frac{n\pi y_1}{b} - \cos\frac{n\pi}{2} - (-1)^n\cos\frac{n\pi}{2} + (-1)^n\cos\frac{n\pi y_2}{b}\Big] .
\end{aligned}
\tag{58}
$$

The unit modal control moments for the first colocated segmented sensor/actuator are defined as

$$
M_{xmn} = r_1^a d_{31} Y_p \mathcal{G}\left[4S_{mn}(1-\cos\frac{m\pi}{2})(1-\cos\frac{n\pi}{2})\right] , \tag{59}
$$

$$
M_{ymn} = r_2^a d_{32} Y_p \mathcal{G}\left[4S_{mn}(1-\cos\frac{m\pi}{2})(1-\cos\frac{n\pi}{2})\right] \tag{60}
$$

Note that the modal feedback factor $\dot{M}_{mn}$ will not vanish except for quadruples of m or n modes. When n = m = 1 or n = m = 3, $\dot{M}_{mn}$'s are identical to those calculated by Eq.(46) for a single piece actuator. Thus, the analysis suggests that the segmented actuator design improves the controllability for even modes without degrading the control merits for odd modes. Since lower modes are generally more important than higher modes, only several lower modes are considered in this study. Detailed parametric study and time-history analyses of the plate will be presented in Part-2 of the paper.

CONCLUSIONS

This paper is concerned with a performance evaluation of distributed, one-piece fully and multi-piece segmented, distributed piezoelectric sensors/actuators. The theoretical analyses suggested that:

1) A single-piece symmetrical distributed sensor layer has sensing deficiencies, observation deficiencies, for all even modes because the locally generated positive and negative charges could be canceled out on the whole effective sensor surface.
2) A single-piece symmetrical distributed actuator layer is also ineffective for controlling all even modes, controllability deficiency, due to similar reasons stated in item 1. However, the charge/voltage is fed back (or injected) to the distributed actuator layer in control applications.
3) Quarterly segmented sensors and actuators can sense and control most of the natural modes, except the quadruple modes, of the plate. The sensing and control effects for all odd modes are identical to the single-piece sensor/actuator configuration.

The analyses showed that segmenting distributed sensor and actuator layers into a number of sub-segments does improve the observability and controllability of the system. The segmented actuator design improves the observability/controllability for even modes without degrading the control merits for all odd modes of a simply supported plate. In general, lower modes are more important than higher modes in structural monitoring and control. Thus, only several lower modes are considered in this study, although further segmentation of actuators are possible and might provide better structural observability/controllability.

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